

Let's start today with statistics – mean, median, mode, range and standard deviation. The topics are simple but the fun lies in the questions. Some questions on these topics can be extremely tricky especially those dealing with median, range and standard deviation. Anyway, we will tackle mean today.

So what do you mean by the arithmetic mean of some observations? I guess most of you will reply that it is the 'Sum of Observations/Total number of observations'. But that is how you *calculate* mean. My question is 'what *is* mean?' Loosely, arithmetic mean is the number that represents all the observations. Say, if I know that the mean age of a group is 10, I would guess that the age of Robbie, who is a part of that group, is 10. Of course Robbie's actual age could be anything but the best guess would be 10.

Say, I tell you that the average age of a group of 10 people is 15 yrs. Can you tell me the sum of the ages of all 10 people? I am sure you will say that it is $10 \times 15 = 150$. You can think of it in two ways:

Mean = Sum of all ages/No of people

So Sum of all ages = Mean * (No of people) = 15×10

Or

Since there are 10 people and each person's age is represented by 15, the sum of their ages = 10×15 . Basically, the total sum was distributed evenly among the 10 people and each person got 15 yrs.

Now, let's say you made a mistake. A boy whose age you thought was 20 was actually 30. What is the correct mean? Again, you can think of it in two ways:

New sum = $150 + 10 = 160$

New average = $160/10 = 16$

Or

You can say that there is an extra 10 that has to be distributed evenly among the 10 people, so each person gets 1 extra. Hence, the average becomes $15 + 1 = 16$.

As you might have guessed, we will work on the second interpretation. Let's look at an example now.

Example 1: The average age of a group of n people is 15 yrs. One more person aged 39 joins the group and the new average is 17 yrs. What is the value of n ?

- (A) 9
- (B) 10
- (C) 11
- (D) 12
- (E) 13

Solution: First tell me, if the age of the additional person were 15 yrs, what would have happened to the average? The average would have remained the same since this new person's age would have been the same as the age that represents the group. But his age is $39 - 15 = 24$ more than the average. We know that we need to evenly split the extra among all the people to get the new average. When 24 is split evenly among all the people (including the new guy), everyone gets 2 extra (since average age increased from 15 to 17). There must be $24/2 = 12$ people now (including the new guy) i.e. n must be 11 (without including the new guy).

Let's look at another similar example though a little trickier. Try solving it on your own first. If not logically, try using the formula approach. Then see how elegant the solution becomes once you start 'thinking' instead of just 'calculating'.

Example 2: When a person aged 39 is added to a group of n people, the average age increases by 2. When a person aged 15 is added instead, the average age decreases by 1. What is the value of n ?

- (A) 7
- (B) 8
- (C) 9
- (D) 10
- (E) 11

Solution: What is the first thing you can say about the initial average? It must have been between 39 and 15. When a person aged 39 is added to the group, the average increases and when a person aged 15 is added, the average decreases.

Let's look at the second case first. When the person aged 15 is added to the group, the average becomes (initial average $- 1$). If instead, the person aged 39 were added to the group, there would be $39 - 15 = 24$ extra which would make the average = (initial average $+ 2$). This difference of 24 creates a difference of 3 in the average. This means there must have been $24/3 = 8$ people (after adding the extra person). The value of n must be $8 - 1 = 7$.

If you use the formula instead, it would take you quite a while to manipulate the two variables to get the value of n . I hope you see the beauty of this method. Next week, we will discuss some GMAT questions based on Arithmetic mean!